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Properties of nonadiabatic superconducting systems with paramagnetic impurity

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Abstract

We present the basic system of equations for a theory of superconductivity for systems with chaotically distributed paramagnetic impurities of substitution in which the Migdal theorem is violated (it cannot be supposed $\omega_0 \ll E_F$). We take into account electron–phonon and impurity diagrams as well as supplementary ones corresponding to the intersection of electron–phonon and electron–impurity lines. Decrease of the quantity T_C with increase of impurity concentration is shown to weaken in comparison with the case of the usual superconductors. The degree of this decrease is determined by quantities m and q_c. The coefficient of the isotope effect α, energy gap and order parameter at $T = 0$ are also calculated. The behaviour of these quantities as a function of impurity concentration depends on the Migdal parameter $m = \omega_0/E_F$ and transferred momentum q_c.

1. Introduction

Recently we have clearly seen the development of the theory with a phonon-mediated pairing mechanism in some papers. This fact is due to the ability to describe the high values of the temperature of the superconducting transition T_C in MgB_2 ($T_C \approx 40$ K) [1] and in fullerenes [2], as well as the experimental confirmation of the considerable influence of the interaction on the dynamics of electrons in oxide superconductors [3]. These factors reveal the great importance of the electron–phonon interaction.

Among the factors playing an important role in obtaining high values of T_C we can point out the following: strong electron–phonon interaction [4, 5], the presence of van Hove singularities or of flat areas in the electron energy spectrum [6, 7], overlapping of energy bands on the Fermi surface (layered structure) [8, 9] and others. In addition the oxide ceramics, fullerenes and organic superconductors are systems with strong electron correlations and low values of Fermi energy. Both these factors have ordinarily not enforced the superconductivity. Nevertheless, there is a necessity to go beyond the Migdal–Eliashberg theory and to build

a theory of superconductivity for nonadiabatic systems in which Fermi energy and Debye energy are comparable as well as to account for the presence in the system of strong electron correlations. There are a large number of papers considering this (see for example, [10–21]). In such systems we have to take into consideration vertex and 'intersecting' diagrams over electron–phonon interactions in mass operators of Green functions which in essence correspond to the account of additional multiparticle effects.

To our mind very interesting results were obtained in [11, 12] for the case of pure nonadiabatic materials. The main effects are due to the vertex corrections and the cross diagrams that show a complex behaviour with respect to the exchanged momentum (q) and frequency (ω). Positive corrections and a corresponding enhancement of T_C arise naturally if the electron–phonon scattering is characterized by small q values. It is possible to have enhancements or reductions of T_C depending on the specific material properties. Vertex corrections and the cross diagram show a similar behaviour with respect to q and ω. For this reason it is useful to introduce the cut-off q_c for the electron–phonon interaction. The value of q_c is assumed to be small because of the effect of strong electron correlations on electron–phonon interactions. This idea is corroborated by simple models [11, 12] and is confirmed by the study of the electron–phonon interaction in such systems [22, 23]. The account of the effects of nonadiabaticity has been shown to produce an essential renormalization of the electron–phonon coupling λ in the Eliashberg equations [24] as well as an increase of the temperature of the superconducting transition at low values of the cut-off momentum q_c in the electron–phonon interaction. At $\lambda \approx 0.5$–1 with the effects of nonadiabaticity, the quantity T_C achieves values equal to the values corresponding to $\lambda \approx 3$ in the Eliashberg equations with strong electron–phonon coupling.

Recent studies of nonadiabatic superconductors demonstrate the significance of the influence of impurity (nonmagnetic and magnetic) on thermodynamic, kinetic and other properties. Investigation of the role of impurity can clarify the nature of high-T_C superconductivity. In this framework the influence of the magnetic chaotically distributed impurity on the temperature of a superconducting transition, the isotope effect and the energy gap is attracting researchers' attention. Nowadays the Abrikosov–Gor'kov's theory [25] (see also [26]) is widely used to describe superconductivity in the nonadiabatic (including high-T_C) systems with magnetic impurity. This theory is based on the adiabatic BCS–Bogolyubov theory of superconductivity presupposing the following condition: the Debye energy is much less than the Fermi energy ($\omega_0 \ll E_F$). This condition is met in ordinary superconductors. Under the Migdal theorem [27] we can neglect all the vertex corrections in mass operators for the Green functions. In the fullerenes, oxides and organic superconductors the Fermi energy and Debye energy can be quantities of the same order [28, 29] and therefore we have to take into consideration the additional multiparticle effects.

The main purposes of this paper are to study the influence of a magnetic impurity on the properties of nonadiabatic superconductors ($\omega_0 \leqslant E_F$) and to answer the question: to what degree is the Abrikosov–Gor'kov theory [25] valid in describing the suppression of superconductivity in high-T_C materials and other systems with strong electron correlations, in which the Debye energy and Fermi energy are comparable.

This paper is organized in the following way. Section 2 discusses the Green functions and the mass operators for the nonadiabatic systems with a paramagnetic impurity. Section 3 presents the equation for the temperature of the superconducting transition T_C and the expression for the coefficient of an isotope effect as well as the analytical expressions in the field of low and high values of impurity concentrations. Section 4 contains the equations obtained for the order parameter Δ at $T = 0$ and the expression for the energy gap Ω_g. Section 5 is devoted to numerical calculations and the conclusion.

2. Green functions and mass operators

The system considered is described by the Hamiltonian

$$H = H_0 + \sum_{\sigma} \int \mathrm{d}\vec{x}\, \psi_{\sigma}^{+}(\vec{x})\psi_{\sigma}(\vec{x})\varphi(\vec{x}) + \sum_{\alpha,\beta} \int \mathrm{d}\vec{x}\, \psi_{\alpha}^{+}(\vec{x})V_{\alpha\beta}(\vec{x})\psi_{\beta}(\vec{x}) \qquad (1)$$

where H_0 is the Hamiltonian of free electrons and phonons, the second term corresponds to the electron–phonon interaction and the third term to the interaction between electrons and the magnetic impurity. $\psi_{\alpha}(\vec{x})$ is the annihilation operator of the electron with spin α in position $\vec{x}$, $\varphi(\vec{x})$ is the phonon operator:

$$V_{\alpha\beta}(\vec{x}) = \sum_{n} V_{\alpha\beta}(\vec{x} - \vec{R}_n) = V_1(\vec{x})\delta_{\alpha\beta} + \tfrac{1}{2}\vec{S}\vec{\sigma}_{\alpha\beta}V_2(\vec{x}), \qquad (2)$$

$\vec{\sigma}$ is the spin-matrix vector and V_1 and V_2 are the scattering potentials on the impurity.

This Hamiltonian coincides formally with the Froelich Hamiltonian when a magnetic impurity is taken into account. Nevertheless, in this Hamiltonian the presence of strong electron correlations induced by the Coulomb interaction are taken into account. It is performed by cutting off the electron–phonon and impurity interactions in the momentum space under the low values of the transferred momentum q_c and q_{c1}, respectively. This approach for the electron–phonon interaction was used in [11, 12] and was based on the studies of the influence of strong electron correlations on the electron–phonon interaction [22]. Low values of q_c promote both positive values of the vertex functions and a dramatic rise in the temperature of the superconducting transition.

First, we introduce the electron and the phonon temperature (Matsubara's) Green functions:

$$\begin{aligned} G_{\beta\alpha}(xx') &= -\langle T\psi_{\beta}(x)\psi_{\alpha}^{+}(x')\rangle \qquad & \tilde{F}_{\alpha\alpha'}(xx') &= -\langle T\psi_{\alpha}^{+}(x)\psi_{\alpha'}^{+}(x')\rangle \\ F_{\beta\beta'}(xx') &= -\langle T\psi_{\beta}(x)\psi_{\beta'}(x')\rangle \qquad & D(xx') &= -\langle T\varphi(x)\varphi(x')\rangle. \end{aligned} \qquad (3)$$

Here $x = (\vec{x}, \tau)$ and $\psi_{\alpha}^{+}(x)$, $\psi_{\alpha}(x)$ are the Heisenberg operators of the particles with the spin α in position x, where the notation $\langle\cdots\rangle$ implies averaging over the states of the system of interacting particles. Then, we study the joint influence of the electron–phonon and the electron–impurity interactions on the electron Green functions (3). To do this we go in these functions to the interaction's representation and use perturbation theory [30]. Next we write the series of the perturbation theory for the Green functions (3), perform averaging over positions $\vec{R}_n$ of the chaotically distributed magnetic impurity and over the orientations of their spins $\vec{S}$ with the help of the relationship

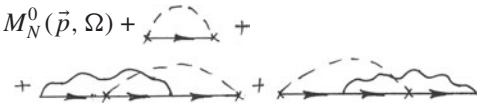
$$\overline{V_{\alpha_1\beta_1}(\vec{x}_1)V_{\alpha_2\beta_2}(\vec{x}_2)} = \frac{c}{V}\sum_{\vec{q}} \mathrm{e}^{-\mathrm{i}\vec{q}(\vec{x}_1-\vec{x}_2)}\left[|V_1(\vec{q})|^2 + \frac{1}{12}S(S+1)|V_2(\vec{q})|^2 \sum_{i} \sigma^{i}_{\alpha_1\beta_1}\sigma^{i}_{\alpha_2\beta_2}\right]. \qquad (4)$$

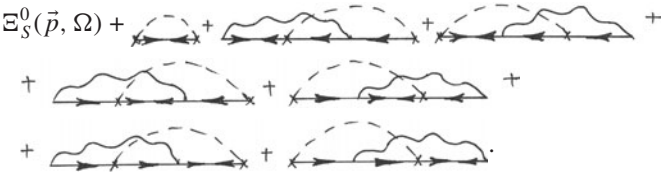
Here c is the impurity concentration. Insert (4) into the above-mentioned series of the perturbation theory for the electron Green functions and perform a summation over spin variables. In this way we obtain the system of equations for the Green functions G and F. We present their values near the temperature of the superconducting transition ($T \sim T_C$) in the $\vec{k}\Omega$ representation:

$$\bar{G}(\vec{p},\Omega) = \frac{1}{\mathrm{i}\Omega - \varepsilon_{\vec{p}} - M_N(\vec{p},\Omega)}; \qquad \bar{F}(\vec{p},\Omega) = \bar{G}(-\vec{p},-\Omega)\Xi_s(\vec{p},\Omega)\bar{G}(\vec{p},\Omega) \qquad (5)$$

where $M_N(\vec{p},\Omega)$ and $\Xi_S(\vec{p},\Omega)$ include the electron–phonon and electron–impurity interactions.

In the diagrams below, expressions for these self-energies can be given in the form

$M_N(\vec{p},\Omega) = M_N^0(\vec{p},\Omega) +$  (6)

$\Xi_S(\vec{p},\Omega) = \Xi_S^0(\vec{p},\Omega) +$  (7)

Here $M_N^0(\vec{p},\Omega)$ and $\Xi_S^0(\vec{p},\Omega)$ contain diagrams corresponding to the electron–phonon interaction (including all possible 'intersecting' lines) [12]. The full lines in M_N^0 and Ξ_S^0 and also in (6) and (7) represent the complete Green function averaging over the positions of the chaotically distributed impurity and the orientations of their spins. The wavy line refers to the electron–phonon interaction and the broken line to the electron–impurity interaction. So, self-energies contain the diagrams with the intersection of the line of electron–phonon interaction both with the line of electron–phonon and electron–impurity interactions as well as the usual electron–phonon and electron–impurity diagrams.

The advantage of the method of anomalous propagators is that the diagrammatic scheme is defined only from the point of view of the self-energy. In this way [12, 20] the Ward identities are automatically satisfied and the eventual inclusion of higher-order diagrams is straightforward. As to the Ward identities the inclusions of vertex corrections in the gap equation implies the inclusion of the same corrections in the self-energy (see equations (1) and (6) in [20]). Here the same situation takes place including only the first nonadiabatic contributions.

In the expressions (6) and (7) we omit the diagrams containing the intersection of electron–impurity lines because their contribution is known to be much less than the contribution from the diagrams with noncrossing electron–impurity lines [30]. The contribution of the crossing diagrams is of order $1/lp_F \ll 1$ (l is the mean free path).

Here we use the model representation for the coupling of the electron–phonon interaction and the electron–impurity interaction:

$$|g_{\vec{p}\vec{p}_1}|^2 = g^2\left(\frac{2k_F}{q_c}\right)^2\theta(q_c - |\vec{p}-\vec{p}_1|) \tag{8}$$

$$W_\pm(\vec{p}-\vec{p}_1) = W_\pm\left(\frac{2k_F}{q_{c1}}\right)^2\theta(q_{c1} - |\vec{p}-\vec{p}_1|) \tag{9}$$

where

$$W_\pm = c[V_1^2 \pm \tfrac{1}{4}S(S+1)V_2^2].$$

The coupling of the electron–phonon interaction $g_{pp'}$ (8) is chosen in accordance with the theory [11, 12] (see also [22, 23]) in which q_c is a small quantity ($q_c \ll 2k_F$). In [22, 23] it was shown that for small hole doping δ strong Coulomb correlations renormalize the electron–phonon interaction, giving rise to the strong forward scattering peak, while the backward scattering is strongly suppressed. This renormalization of the coupling of the electron–phonon interaction is described by the vertex function (8), where g is the bare coupling constant and the residual part of (8), forming the vertex function, is strongly peaked at $\vec{q} = 0$. The physical meaning of this renormalization is that each quasiparticle due to the suppression of the double occupancy on the same lattice-strong correlations, is surrounded by a giant correlation hole

with characteristic size $R \sim a/\delta$, where a is the lattice constant. In [11, 12] it was shown that in the presence of the pronounced forward electron–phonon interaction the vertex corrections at low values of q result in positive values of the vertex functions and high values of the temperature of the superconducting transition.

Strong electron correlations undoubtedly affect the electron–impurity interaction in a similar manner to the electron–phonon interaction (9), where $W_\pm$ is the bare scattering impurity potential and the residual part of (9), forming the vertex function, is strongly peaked at $\vec{q} = 0$ because the quasiparticle is surrounded by a giant correlation hole. But the measure of the localization (renormalization) of the impurity potential differs qualitatively from the measure of the electron–phonon interaction. So we assume here the momentum cut-off $q_{c1} \neq q_c$ for the sake of generality. The difference of q_{c1} from q_c may be unimportant because of their smallness.

The first step in the solution of the gap equation is to perform an average over momenta. For the adiabatic case it is consistent to perform this average over the Fermi surface. But in the nonadiabatic case the average over momenta differs from the average over the Fermi surface only in the higher-order nonadiabaticity corrections [12]. Therefore, the momentum dependence of the self-energies (6) and (7) is eliminated, leaving only their dependence on the frequencies. Hereafter, $\langle\langle \cdots \rangle\rangle_{FS}$ means the average over the Fermi surface.

In terms of definitions (8) and (9) the expressions for self-energies $M_N(\vec{p}, \Omega)$ and $\Xi_S(\vec{p}, \Omega)$, averaged over the Fermi surface, can be presented in the following form:

$$\langle\langle M_N(\vec{p}, \Omega)\rangle\rangle_{FS} = M_N(\Omega) = \frac{1}{\beta V} \sum_{\vec{p}_1, \Omega_1} \bar{V}_N(\Omega, \Omega_1) \bar{G}(\vec{p}_1, \Omega_1) + \frac{1}{V} \sum_{\vec{p}_1} \bar{W}_+ \bar{G}(\vec{p}_1, \Omega) \tag{10}$$

$$\langle\langle \Xi_S(\Omega)\rangle\rangle_{FS} = \Xi_S(\Omega) = \frac{1}{\beta V} \sum_{\vec{p}_1, \Omega_1} \bar{V}_s(\Omega, \Omega_1) \bar{F}(\vec{p}_1, \Omega_1) + \frac{1}{V} \sum_{\vec{p}_1} \bar{W}_- \bar{F}(\vec{p}_1, \Omega) \tag{11}$$

where the effective electron–phonon $\bar{V}_N$, $\bar{V}_S$ and electron–impurity $\bar{W}_\pm$ couplings, including the first nonadiabatic contributions, are given by

$$\bar{V}_N(\Omega\Omega_1) = -g^2 D(\Omega\Omega_1)[1 + \lambda \bar{P}_V(Q_c, \Omega\Omega_1)] \tag{12}$$

$$\bar{V}_S(\Omega\Omega_1) = -g^2 D(\Omega\Omega_1)[1 + 2\lambda \bar{P}_V(Q_c, \Omega\Omega_1) + \lambda \bar{P}_c(Q_c, \Omega\Omega_1)] - \lambda D(\Omega\Omega_1) \times [\Gamma_c^I(Q_c Q_{c1}, \Omega\Omega_1) + \Gamma_c^{II}(Q_c Q_{c1}, \Omega\Omega_1) + \Gamma_V(Q_c Q_{c1}, \Omega\Omega_1)] \tag{13}$$

$$\bar{W}_\pm = W_\pm[1 + 2\lambda \bar{P}_V(\Omega\Omega)]; \qquad \bar{P}_V(\Omega\Omega) = \bar{P}_V(Q_c, \Omega\Omega_1)|_{\Omega_1 = \Omega}. \tag{14}$$

Here $\bar{P}_V$, $\bar{P}_c$ and Γ_V, Γ_c are the vertex corrections to the self-energies, averaged over the Fermi surface:

$$\bar{P}_V(Q_c, \Omega\Omega_1) = -\frac{1}{\beta V} \left\langle\!\!\left\langle \sum_{\vec{p}_2, \Omega_2} \left(\frac{2k_F}{q_c}\right)^2 \frac{1}{N_0} \theta(q_c - |\vec{p} - \vec{p}_2|) \times \bar{G}(\vec{p}_2, \Omega_2) \bar{G}(\vec{p}_1 + \vec{p}_2 - \vec{p}, \Omega_1 + \Omega_2 - \Omega) D(\Omega, \Omega_2) \right\rangle\!\!\right\rangle_{FS} \tag{15}$$

$$\bar{P}_c(Q_c, \Omega\Omega_1) = -\frac{1}{\beta V} \left\langle\!\!\left\langle \sum_{\vec{p}_2, \Omega_2} \left(\frac{2k_F}{q_c}\right)^2 \frac{1}{N_0} \theta(q_c - |\vec{p} - \vec{p}_2|) \times \bar{G}(\vec{p}_2, \Omega_2) \bar{G}(\vec{p}_2 - \vec{p} - \vec{p}_1, \Omega_2 - \Omega - \Omega_1) D(\Omega, \Omega_2) \right\rangle\!\!\right\rangle_{FS} \tag{16}$$

$$\Gamma_V(Q_cQ_{c1},\Omega\Omega_1)=W_+\left\langle\!\!\left\langle\sum_{\vec{p}_2}\left(\frac{2k_F}{q_{c1}}\right)^2\frac{1}{N_0V}\theta(q_{c1}-|\vec{p}-\vec{p}_2|)\right.\right.$$
$$\left.\left.\times\bar{G}(\vec{p}_1+\vec{p}_2-\vec{p},\Omega_1)\bar{G}(\vec{p}_2,\Omega)\right\rangle\!\!\right\rangle_{FS} \tag{17}$$

$$\Gamma_c^I(Q_cQ_{c1},\Omega\Omega_1)=W_-\left\langle\!\!\left\langle\sum_{\vec{p}_2}\left(\frac{2k_F}{q_c}\right)^2\frac{1}{N_0V}\theta(q_c-|\vec{p}-\vec{p}_2|)\right.\right.$$
$$\left.\left.\times\bar{G}(\vec{p}_2,\Omega_1)\bar{G}(\vec{p}_2-\vec{p}_1-\vec{p},-\Omega)\right\rangle\!\!\right\rangle_{FS}. \tag{18}$$

Γ_c^{II} is obtained from Γ_c^I by substituting $q_c\leftrightarrow q_{c1}$ and $\Omega_1\leftrightarrow\Omega$ ($Q_c=q_c/2k_F$, $Q_{c1}=q_{c1}/2k_F$).

Note that (10), (11) and the quantities which enter into them contain the complete Green functions, taking into account the electron–phonon and electron–impurity interactions in all orders of perturbation theory. The approximation consists in accounting for only linear terms with respect to nonadiabaticity (diagrams with crossing of two electron–phonon lines or of electron–phonon and electron–impurity lines).

For the phonon propagator $D(\Omega,\Omega_1)$ the expression corresponding to the Einstein spectrum ω_0 is chosen:

$$D(\Omega,\Omega_1)=-\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}. \tag{19}$$

To calculate quantities (15)–(18) we apply the method developed in [11, 12]: we choose the linear dispersion law of the electron energy and consider the low values of q_c, q_{c1}. These assumptions permit us to perform integration over frequency Ω_2, energy $\varepsilon_{\vec{p}_2}$ and angle variables with the following averaging over the Fermi surface. In the weak coupling approximation $T_C/\omega_0\ll 1$ we obtain

$$\begin{aligned}\bar{P}_V(Q_c\Omega\Omega_1)&=P_V(Q_c\Omega\Omega_1)+\mathrm{O}\left(\frac{W_+}{\omega_0}\right)\\ \bar{P}_c(Q_c\Omega\Omega_1)&=P_c(Q_c\Omega\Omega_1)+\mathrm{O}\left(\frac{W_-}{\omega_0}\right)\end{aligned} \tag{20}$$

where P_V, P_c are the quantities calculated in [12]. Because $W_\pm/\omega_0\sim W_\pm/E_F\ll 1$, we can neglect the difference of the quantities (20) from P_V and P_c. Here we give their values at $\Omega=0$ and $\Omega_1=\omega_0$:

$$\begin{aligned}P_V(Q_c,0,\omega_0)=P_V(m,Q_c)&=-\varphi(m)+\left[\frac{\pi}{4}-\arctan\frac{m}{1+m}+\varphi(m)\right]\\ &\times\frac{m^2}{4Q_c^4}\left\{\sqrt{\eta_c(m)}-1-\ln\frac{1+\sqrt{\eta_c(m)}}{2}\right\}\\ P_c(Q_c,0,\omega_0)=P_c(m,Q_c)&=-\varphi(m)+\left[\frac{\pi}{4}-\arctan\frac{m}{1+m}+\varphi(m)\right]\\ &\times\frac{m}{4Q_c^2(1-Q_c^2/2)}\arctan\frac{4Q_c^2(1-Q_c^2/2)}{m}\end{aligned} \tag{21}$$

where

$$\varphi(m) = m(1+m)\frac{(1+m)^2+2m^2}{[(1+m)^2+m^2]^2}; \qquad \eta_c(m) = 1+\left(\frac{4Q_c^2}{m}\right)^2$$
$$P_V(\Omega\Omega)|_{\Omega=0} = -\frac{m}{1+m}. \tag{22}$$

Here $m = 2\omega_0/E_F$ is the Migdal parameter.

For the quantities Γ_V and Γ_c caused by the joint electron–phonon and impurity scattering we obtain

$$\Gamma_V(Q_cQ_{c1},\Omega\Omega_1) = W_+\left\{L(\Omega\Omega_1)+[K(\Omega\Omega_1)-(\Omega-\Omega_1)^2L(\Omega\Omega_1)]\frac{1}{4E_F^2Q_c^2Q_{c1}^2}\right.$$
$$\left.\times\left[\sqrt{1+\left(\frac{2E_FQ_cQ_{c1}}{\Omega-\Omega_1}\right)^2}-1-\ln\frac{1}{2}\left(1+\sqrt{1+\left(\frac{2E_FQ_cQ_{c1}}{\Omega-\Omega_1}\right)^2}\right)\right]\right\} \tag{23}$$

$$\Gamma_c^I(Q_cQ_{c1},\Omega\Omega_1) = W_-\left\{L(-\Omega_1\Omega)+\left[\frac{K(-\Omega_1\Omega)}{\Omega+\Omega_1}-(\Omega+\Omega_1)L(-\Omega_1\Omega)\right]\right.$$
$$\left.\times\frac{1}{2E_FQ_c^2Q_{c1}^2}\int_0^{Q_{c1}}\frac{Q\,\mathrm{d}Q}{1-Q^2}\arctan\frac{4Q_c^2E_F(1-Q^2)}{\Omega+\Omega_1}\right\} \tag{24}$$

where

$$K(\Omega\Omega_1) = 2(\Omega-\Omega_1)\left[\arctan\frac{E_F}{\Omega}-\arctan\frac{E_F}{\Omega_1}\right]$$
$$L(\Omega\Omega_1) = -\frac{E_F}{E_F^2+\Omega_1^2}+\frac{\Omega-\Omega_1}{\Omega_1^3}\frac{E_F}{[1+(E_F/\Omega_1)^2]^2}. \tag{25}$$

Furthermore, in (10) and (11) we perform conventionally the integration over the energy in the limits $-E_F < \varepsilon_{\vec{p}_1} < E_F$ and introduce the definitions

$$\tilde{\Omega} = \tilde{\Omega}(\Omega) = \Omega - \operatorname{Im} M_N(\Omega)$$
$$= \Omega+\frac{\pi N_0}{\beta}\sum_{\Omega_1}\bar{V}_N(\Omega\Omega_1)\frac{\tilde{\Omega}_1}{|\tilde{\Omega}_1|}\frac{2}{\pi}\arctan\frac{E_F}{|\tilde{\Omega}_1|}+\pi N_0\bar{W}_+\frac{\tilde{\Omega}}{|\tilde{\Omega}|}\frac{2}{\pi}\arctan\frac{E_F}{|\tilde{\Omega}|} \tag{26}$$

$$\Xi(\Omega) = \tilde{\Delta}(\Omega) = \bar{\Delta}+\pi N_0\bar{W}_-\frac{\tilde{\Delta}(\Omega)}{|\tilde{\Omega}|}\frac{2}{\pi}\arctan\frac{E_F}{|\tilde{\Omega}|} \tag{27}$$

where

$$\bar{\Delta} = \frac{\pi N_0}{\beta}\sum_{\Omega_1}\bar{V}_S(\Omega\Omega_1)\frac{\tilde{\Delta}(\Omega_1)}{|\tilde{\Omega}_1|}\frac{2}{\pi}\arctan\frac{E_F}{|\tilde{\Omega}_1|}. \tag{28}$$

3. Temperature of superconducting transition and isotope effect

The self-consistent system of equations (26)–(28) is analysed here to clarify the joint influence of both the nonadiabaticity effect and the paramagnetic impurity on the temperature of the superconducting transition T_C and the isotope coefficient α. To this end we make some simplifications. In (12)–(14) we put $\Omega = 0$, $\Omega_1 = \omega_0$ in square brackets, introduce the Migdal parameter $m = 2\omega_0/E_F$ and consider the weak coupling approximation $T_C/\omega_0 \ll 1$. Then we further introduce the following notations:

$$\lambda_\Delta = \lambda[1+2\lambda P_V(m,Q_c)+\lambda P_c(m,Q_c)+\Gamma_c^I(Q_cQ_{c1},m)$$
$$+\Gamma_c^{II}(Q_cQ_{c1},m)+2\Gamma_V(Q_cQ_{c1},m)] \tag{29}$$

$$\lambda_z = \lambda[1+\lambda P_V(m,Q_c)]; \qquad \bar{W}_\pm = W_\pm[1+2\lambda P_V(0,0)]. \tag{30}$$

On the basis of (23)–(25), $\Gamma_{V,c}$ is easily seen to be of the order $\sim W_{\pm}/\omega_0 \ll 1$, which allows us to further neglect these terms in (29), considering the quantity $\lambda_\Delta = \lambda^0_\Delta$, irrespective of the impurity concentration. Taking into account this fact and definitions (29) and (30), equations (26)–(28) can have the following form:

$$\begin{aligned}
\bar{\Delta}(\Omega) &= \frac{\pi}{\beta}\lambda^0_\Delta \sum_{\Omega_1} \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \frac{\tilde{\Delta}(\Omega_1)}{|\tilde{\Omega}_1|} \frac{2}{\pi} \arctan \frac{E_F}{|\tilde{\Omega}_1|} \\
\tilde{\Delta}(\Omega) &= \bar{\Delta}(\Omega) + \pi N_0 \bar{W}_- \frac{\tilde{\Delta}(\Omega)}{|\tilde{\Omega}|} \frac{2}{\pi} \arctan \frac{E_F}{|\tilde{\Omega}|} \\
\tilde{\Omega} &= \Omega Z_0 + \pi N_0 \bar{W}_+ \frac{2}{\pi} \arctan \frac{E_F}{\tilde{\Omega}}
\end{aligned} \tag{31}$$

where

$$Z_0 = 1 + \frac{2\pi}{\beta\Omega}\lambda_z \sum_{\Omega_1} \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \arctan \frac{E_F}{|\tilde{\Omega}_1|} = 1 + \frac{\lambda_z}{1+m}; \qquad \lambda^0_\Delta = \lambda_\Delta|_{c=0}. \tag{32}$$

Introducing the quantity $u = \tilde{\Omega}/\tilde{\Delta}$ and presenting the last two equations (31) and (32) in the form

$$u(\Omega) = \frac{\Omega Z_0}{\bar{\Delta}} + \frac{\pi N_0}{\bar{\Delta}}[\bar{W}_+ - \bar{W}_-]\frac{2}{\pi} \arctan \frac{E_F}{\tilde{\Omega}} \tag{33}$$

we also have

$$\bar{\Delta}(\Omega) = \frac{\pi}{\beta}\lambda^0_\Delta \sum_{\Omega_1} \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \frac{1}{|u(\Omega_1)|} \frac{2}{\pi} \arctan \frac{E_F}{|\tilde{\Omega}_1|}. \tag{34}$$

As in the latter case (34), the function under notation of the sum is quickly decreased and we can substitute the quantity $u(\Omega_1)$ by its value in the field of low Ω and thus (34) may have the form

$$\bar{\Delta}(\Omega) = \frac{\pi}{\beta}\lambda^0_\Delta \sum_{\Omega_1} \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \frac{\bar{\Delta}(\Omega_1)}{Z_0\Omega_1 + \pi N_0[\bar{W}_+ - \bar{W}_-]\operatorname{sign}\Omega_1} \frac{2}{\pi} \arctan \frac{E_F}{|\tilde{\Omega}_1|}. \tag{35}$$

Further, in this equation we perform algebraic transformations and approximations similar to those of the adiabatic theory [31] (see also [12, 35]).

After making these calculations and neglecting terms of the order $\Gamma/E_F \ll 1$, we obtain for the temperature of superconducting transition T_C:

$$\ln \frac{T_C}{T_{C0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2}+\rho\right). \tag{36}$$

Here T_{C0} is determined by the following expression:

$$T_{C0} = \frac{2\omega_0\gamma_e}{\sqrt{e}(1+m)\pi} \exp\left[\frac{1}{2}\frac{m}{1+m}\right] \exp\left[-\frac{1+\lambda_z/(1+m)}{\lambda^0_\Delta}\right] \tag{37}$$

where γ_e is Euler's constant. λ_z and λ^0_Δ depend on m and Q_c and ρ are given after formula (38):

$$\begin{aligned}
\rho &= \frac{\Gamma}{2\pi T_C} f_c(m), \qquad f_c(m) = \frac{1}{Z_0}\left(1 - 2\lambda\frac{m}{1+m}\right), \\
\Gamma &= \frac{1}{2\tau_s} = \frac{1}{2} c N_0 \pi S(1+S) V_2^2
\end{aligned} \tag{38}$$

ψ is the digamma function and τ_s is the relaxation time of the scattering of electrons from the magnetic impurity.

Equation (36) coincides in its form with the corresponding equation of the Abrikosov–Gor'kov theory [18, 25]. But T_{C0} and the scattering parameter ρ are redetermined and essentially depend on the nonadiabaticity parameter m and the cut-off momentum Q_c. On the basis of (36) for the critical concentration of the impurity when $T_C = 0$ we obtain

$$\Gamma_{cr} = \frac{\pi T_{C0}}{2\gamma_e f_c(m)}. \tag{39}$$

Because $f_c(m) < 1$, the decrease of T_C with the increase of impurity concentration will be weaker than in ordinary superconductors, and the critical concentration of the impurity will be higher due both to higher values of T_{C0} and to $f_c(m) < 1$. From the definition of the scattering parameter ρ the nonadiabaticity effect ($m \neq 0$) is seen to reduce it. The account of nonadiabaticity results in the renormalization of some parameters of the theory. In particular, these effects significantly increase the coupling of the electron–phonon interaction λ (29) acting as an additional factor of attraction between electrons forming the Cooper pairs. This factor is also revealed in the definition of the scattering parameter ρ and, as follows from (38), it is contained in the quantity ρ ($\lambda \neq 0, m \neq 0$). This leads to the decrease of the scattering parameter ρ. In its turn this weakens the pair destructive influence of the paramagnetic impurity caused by the additional electron–phonon interaction described by diagrams with crossed impurity and phonon lines.

Using (36) for the isotope coefficient $\alpha = -\mathrm{d}\ln T_C/\mathrm{d}\ln M$ we obtain

$$\alpha = \frac{\alpha_0 - \frac{\Gamma}{2\pi T_C}\psi'(1/2+\rho)\frac{\mathrm{d}}{\mathrm{d}\ln m} f_c(m)}{1 - \frac{\Gamma}{2\pi T_C}\psi'(1/2+\rho) f_c(m)} \tag{40}$$

where $\alpha_0 = -\mathrm{d}\ln T_{C0}/\mathrm{d}\ln M$. As $f(m) < 1$, α increases with increasing impurity concentration. At $m = 0$, we obtain the case of ordinary superconductors [33].

Note that equation (36) is derived by assuming a low value of the cut-off momentum of the electron–impurity interaction ($q_{c1} \ll 2p_F$). This choice of the quantity q_{c1} permits us to perform analytical calculations for the impurity vertices $\Gamma_{c,V}$ (17) and (18). T_C is seen from (36) to be independent of the concentration of nonmagnetic impurity because at $S = 0$ we have $T_C = T_{C0}$. This result is in agreement with the Anderson theorem [34] according to which a nonmagnetic impurity does not influence the quantity T_C in isotropic systems. Apparently this result may be considered as a criterion of the valid choice of low values of q_{c1}.

4. Order parameter at $T = 0$ and energy gap

If $T \neq T_C$, the expressions for mass operators (6) and (7) contain the supplementary diagrams with multiplications of two anomalous Green functions. It results in the additional renormalization of the quantities $V_N(\Omega\Omega_1)$ and $V_S(\Omega\Omega_1)$ in the equations used to determine the order parameter as well as the diagrams shown in (6) and (7). Nevertheless, in the weak coupling approximation ($\omega_0 \gg \Delta$) considered here, the contribution of these diagrams is small. So we can assume that the parameters λ_Δ and λ_z will have the same form as in the case of the temperature of the superconducting transition T_C (29) and (30).

Starting from definitions (5)–(7) and after making the operations similar to those in the previous section we give the system of equations for determination of the order parameter Δ in the following form:

$$Z\Delta = \frac{\pi\lambda_\Delta}{\beta}\sum_{\Omega_1}\frac{1}{\sqrt{u^2+1}}\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}\arctan\frac{E_F}{Z\tilde{\Delta}\sqrt{u^2+1}} \tag{41}$$

$$\Delta u = \Omega + \bar{\gamma}\frac{u}{\sqrt{u^2+1}}\frac{2}{\pi}\arctan\frac{E_F}{Z\tilde{\Delta}\sqrt{u^2+1}} \tag{42}$$

where

$$\tilde{\Delta}\sqrt{u^2+1} \approx \Delta\sqrt{u^2+1} + N_0 W_- \frac{1}{\sqrt{u^2+1}} \frac{2}{\pi} \arctan \frac{E_F}{Z\Delta\sqrt{u^2+1}}$$
$$Z(\Omega) \approx 1 + \frac{\lambda_z \pi}{\beta\Omega} \sum_{\Omega_1} \frac{u}{\sqrt{u^2+1}} \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \arctan \frac{E_F}{Z\Delta\sqrt{u^2+1}} \tag{43}$$
$$\bar{\gamma} = \frac{\pi N_0}{Z_0}[\bar{W}_+ - \bar{W}_-] = \Gamma f_c(m).$$

Here we neglect the terms of the order $W_-/E_F \ll 1$ and further we will not take into account terms of the order W_+/E_F and $\bar{\gamma}/E_F$ and $\bar{\gamma}/\omega_0 \ll 1$.

Then in expressions (41)–(43) we go from summation to integration in the usual manner. Further we carry out similar transformations as in the previous section. Then equation (41) has the form:

$$\begin{aligned} 1 = \lambda_\Delta^0 \int_0^\infty & \frac{\omega_0^4}{(\Omega_1^2+\omega_0^2)^2} \frac{\mathrm{d}\Omega_1}{Z\Delta\sqrt{u^2+1}} \frac{2}{\pi} \arctan \frac{E_F}{Z\Delta\sqrt{u^2+1}} \\ &= \lambda_\Delta^0 \int_0^\infty \frac{\omega_0^2}{\Omega_1^2+\omega_0^2} \frac{\mathrm{d}\Omega_1}{Z\Delta\sqrt{u^2+1}} \\ &\quad - \lambda_\Delta^0 \int_0^\infty \frac{\omega_0^2}{\Omega_1^2+\omega_0^2} \frac{\mathrm{d}\Omega_1}{\Delta\sqrt{u^2+1}} \frac{2}{\pi} \arctan \frac{Z\Delta\sqrt{u^2+1}}{E_F} \\ &\quad - \lambda_\Delta^0 \int_0^\infty \frac{\omega_0^2\Omega_1^2}{(\Omega_1^2+\omega_0^2)^2} \frac{\mathrm{d}\Omega_1}{Z\Delta\sqrt{u^2+1}} \frac{2}{\pi} \arctan \frac{E_F}{Z\Delta\sqrt{u^2+1}}. \end{aligned} \tag{44}$$

In the approximation $\bar{\gamma}/E_F \ll 1$ proposed here, equation (42) has the form

$$\Delta u = \Omega + \bar{\gamma} \frac{u}{\sqrt{u^2+1}}. \tag{45}$$

In the first term of equation (44) we go from integration over Ω_1 to integration over u_1, with the help of relationship (45), under which the limits of integration have the following form:

$$u(0) = \begin{cases} 0 & \text{at } \frac{\bar{\gamma}}{\Delta} < 1 \\ \sqrt{\left(\frac{\bar{\gamma}}{\Delta}\right)^2 - 1} & \text{at } \frac{\bar{\gamma}}{\Delta} > 1. \end{cases} \tag{46}$$

In this term we pick out the logarithmic singularity under Δ. In other terms of equation (44) that singularity is absent because, in the weak coupling approximation, $\omega_0 \gg \Delta$ and at $\gamma/E_F \ll 1$ when calculating we can make $\Delta \to 0$ and $\gamma \to 0$.

In this way we obtain

$$\ln \frac{\Delta}{\Delta_0} = -\frac{\bar{\gamma}}{\Delta}\frac{\pi}{4} \qquad \text{at } \frac{\bar{\gamma}}{\Delta} < 1$$
$$\ln \frac{\Delta}{\Delta_0} = -\ln\left[\sqrt{\left(\frac{\bar{\gamma}}{\Delta}\right)^2 - 1} + \frac{\bar{\gamma}}{\Delta}\right] + \frac{\sqrt{(\frac{\bar{\gamma}}{\Delta})^2-1}}{2\frac{\bar{\gamma}}{\Delta}} - \frac{\bar{\gamma}}{2\Delta} \arctan \frac{1}{\sqrt{(\frac{\bar{\gamma}}{\Delta})^2-1}} \tag{47}$$
$$\text{at } \frac{\bar{\gamma}}{\Delta} > 1$$

where

$$\Delta_0 = \frac{2\omega_0}{(m+1)\sqrt{e}} \exp\left(\frac{1}{2}\frac{m}{m+1}\right) \exp\left(-\frac{Z_0}{\lambda_\Delta^0}\right). \tag{48}$$

Z is calculated on the basis of expression (43) in the approximation similar to the way of calculating the last two terms of equation (44), and coincides with formula (32), while $\bar{\gamma}$ is defined by formula (43). Equations (47) at $m = 0$ coincide formally with the corresponding expressions from [25, 26] in ordinary superconductors. In nonadiabatic systems the quantity Δ_0 (48) and the scattering parameter $\bar{\gamma} = \Gamma f_c(m)$ depend essentially on the Migdal parameter m and the cut-off momentum of the phonon interaction Q_c.

To determine the energy gap we consider the density of electron states as in [25, 26] by the relationships

$$N_S(\Omega) = N_0 \operatorname{Re} \frac{u}{\sqrt{u^2 - 1}} \tag{49}$$

$$\Delta u = \Omega + \mathrm{i}\bar{\gamma} \frac{u}{\sqrt{u^2 - 1}} \tag{50}$$

where $N_S(\Omega) = 0$ at $u < 1$. Then we study equation (50) at $u < 1$:

$$\Omega = \Delta u - \bar{\gamma} \frac{u}{\sqrt{1 - u^2}}. \tag{51}$$

The maximum value of Ω at which the density of electron states is equal to zero corresponds to the value of the energy gap Ω_g in the energy spectrum. We find Ω_g and $u(\Omega_g)$ with the help of (51) from the condition $\mathrm{d}\Omega/\mathrm{d}u = 0$:

$$u(\Omega_g) = \left[1 - \left(\frac{\bar{\gamma}}{\Delta}\right)^{2/3}\right]^{1/2}, \qquad \Omega_g = \left[1 - \left(\frac{\bar{\gamma}}{\Delta}\right)^{2/3}\right]^{3/2}. \tag{52}$$

From this equation it follows that, at $\bar{\gamma}/\Delta = 1$, $\Omega_g = 0$, or with the help of (47),

$$\bar{\gamma}_{\Omega_g=0} = \Delta, \qquad \text{or} \qquad \Gamma_{\Omega_g=0} = \frac{\Delta_0(m)}{f_c(m)} \exp\left(-\frac{\pi}{4}\right). \tag{53}$$

The last formula determines the critical concentration of the impurity at which the gapless superconductivity starts. This quantity is essentially dependent on the Migdal parameter m. As $\Delta_0(m) > \Delta_0$ ($m = 0$) and $f_c(m) < f_c$ ($m = 0$), the quantity $\Gamma_{\Omega_g=0}$ at $m \neq 0$ is greater than in ordinary superconductors.

5. Numerical calculations and conclusions

In numerical calculations low values of the transferred momentum ($q_c/2k_F = 0.1$) are chosen in accordance with the results of [11, 12] where the nonadiabaticity effect results in a significant rise of the temperature of the superconducting transition at $q_c \ll 2k_F$. The dependence of T_C/T_{C0} on $\Gamma/2\pi T_0$ ($T_0 = T_{C0}$ at $m = 0$) is shown in figure 1. In figure 1 we can clearly see the slower decrease of T_C/T_{C0} on $\Gamma/2\pi T_0$ with the increase of the non-adiabaticity parameter m from the value $m = 0$ (the case of ordinary superconductors, curve 1) to some definite value $m = 0.085$ (curve 3). Furthermore, on achieving for m the value $m = 1$ (curve 5) a larger decrease of T_C/T_{C0} on $\Gamma/2\pi T_0$ in comparison with the case for $m = 0.085$ (curve 3) and a smaller decrease in comparison with the case for $m = 0$ (curve 1) are observed. This unusual behaviour of the relation T_C/T_{C0} is described by the existence of the maximum (at $m = 0.085$) in the dependence T_{C0} as a function of m (see [11, 35]). The appearance of the maximum in the dependence of the critical impurity concentration on the Migdal parameter is due to this fact.

In figure 2 we illustrate the dependence of the coefficient of the isotope effect as a function of impurity concentration $\Gamma/2\pi T_0$ with the change of the Migdal parameter m. The notation of curves in this figure coincides with the notation in figure 1. We find that α increases with

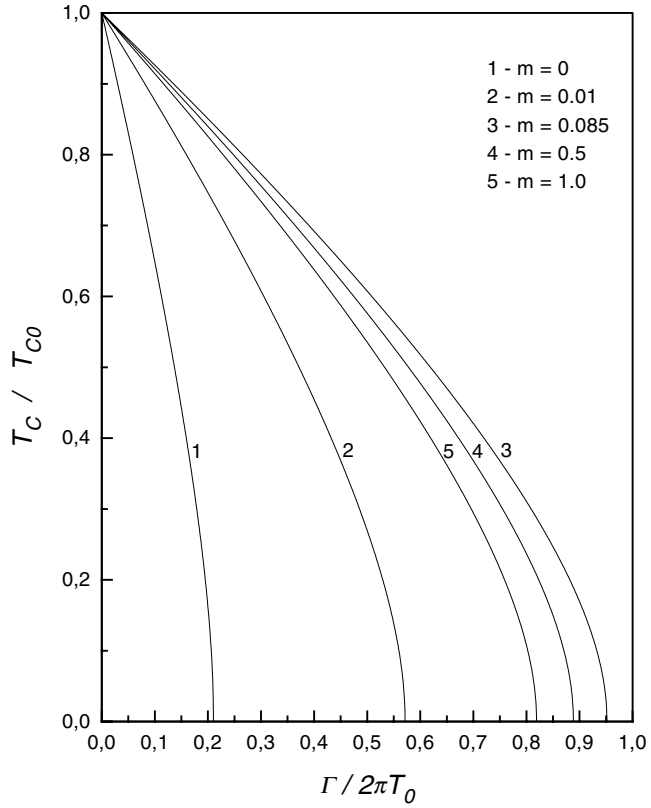


Figure 1. Dependence of T_C/T_{C0} on $\Gamma/2\pi T_0$ at different values of the Migdal parameter $m = 2\omega_0/E_F$.

impurity concentration and decreases with increase in m (curves 2–5). At a higher value of the parameter m (>0.2) the coefficient of the isotope effect α can be less than 0.5 (see, for example, curve 4) for both pure and doped superconductors, depending on the value of the impurity concentration. At the value of $\omega_0 \sim E_F$ (or $m \sim 1$), which occurs in the high-T_C materials, $\alpha < 1/2$ for pure superconductors (curve 5) and it can have values lower or higher than $1/2$ according to the impurity concentration.

We have studied various dependences of the quantities T_C/T_{C0} and the coefficient of the isotope effect α, assuming that the Migdal parameter $m = 2\omega_0/E_F$ changes from 0 to 1 and that the value of the transferred momentum $Q_c \ll 1$. As was noted in [11, 12] a small quantity of Q_c is due to the presence of strong electron correlations in the system. It is interesting to note that lower values of $m \ll 1$ and $Q_c \ll 1$ can result in higher values of T_C. So the highest T_C can be achieved in systems with larger charge carrier density and strong electron correlations. These materials are as yet unknown. Therefore it is interesting to study the field of variation of the quantities $m = 0.5$–1 and $Q_c \ll 1$, inherent to the high-T_C materials. This is shown in the figures given above. To understand the behaviour of the considered quantities in this field we should make comparisons with the case of $m = 0$, which corresponds to the case of ordinary superconductors. From figure 1 we can see how quickly superconductivity is suppressed at different values of m. In the experiment, we have $x_{cr} = 0.01$ in low temperature systems [37], $x_{cr} = 0.04$ in lanthanum ceramics [40] and $x_{cr} = 0.13$–0.15 in yttrium ceramics [36]. These

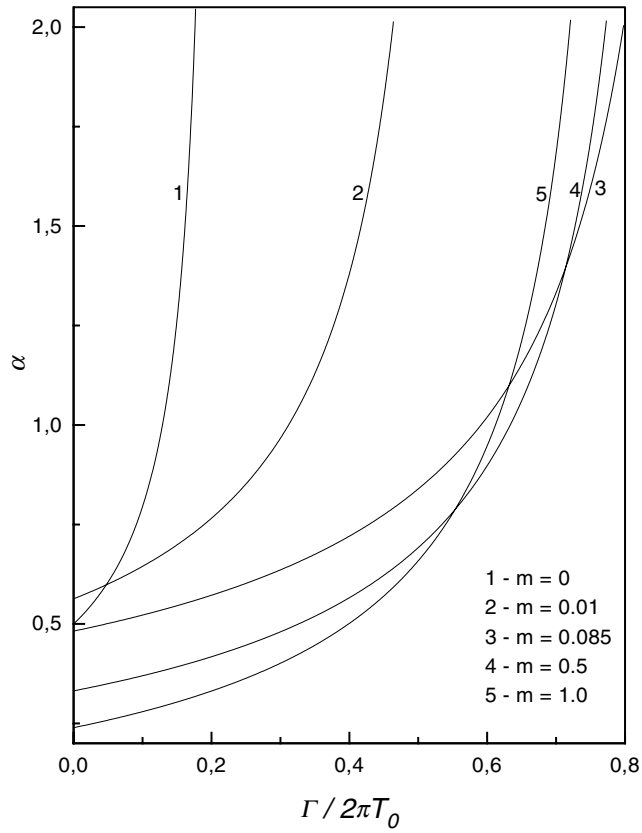


Figure 2. Dependence of the coefficient of the isotope effect α on $\Gamma/2\pi T_0$ at different values of the Migdal parameter $m = 2\omega_0/E_F$.

values of the critical concentration of the impurity x_{cr} are caused by the dependence of x_{cr} on both T_{C0} of the pure substance, which in the papers cited above is changed from $T_{C0} = 7$ K [37] to $T_{C0} = 40$ K [40] and $T_{C0} = 90$ K [36], and on the properties of the basic substance. These experimental data are in qualitative agreement with the theory given above. In this way both nonadiabaticity effects ($m \neq 0$) and strong electron correlations ($Q_c \ll 1$), fixing the other parameters of the given theory characterizing the superconducting state of a substance (λ is the coupling of the electron–phonon interaction, ω_0 is the Einstein frequency and other characteristic quantities), are taken into account.

The studies of the coefficient of the isotope effect α clearly reveal its increase with impurity concentration. In the case of the high-T_C materials ($\omega_D \sim E_F$) α as a function of impurity concentration takes the low values ($\sim$0.2) at low values of impurity concentration and can achieve values of $\sim$1.5 and higher with the increase of impurity concentration. Experimental studies concerning the coefficient of the isotope effect with the change of impurity concentration, while suppressing T_C, result in the increase of α [33], being strongly dependent on the content of the substance. For example, in $La_{2-x}Sr_xCuO_4$ at $x \approx 0.11$ and $T_C = 30$ K the value $\alpha \approx 0.6$ [38] was obtained, in $Y_{1-x}Pr_xBa_2Cu_3O_7$ at $T_C \sim 30$ K $\alpha \approx 0.4$ [39] and in the compounds $La_{1.85}Sr_{0.15}Cu_{1-x}M_x$ α gets the values $0.2 - 1.3$ in accordance with impurity concentration M = Ni, Fe [40]. These experimental studies are in good qualitative agreement

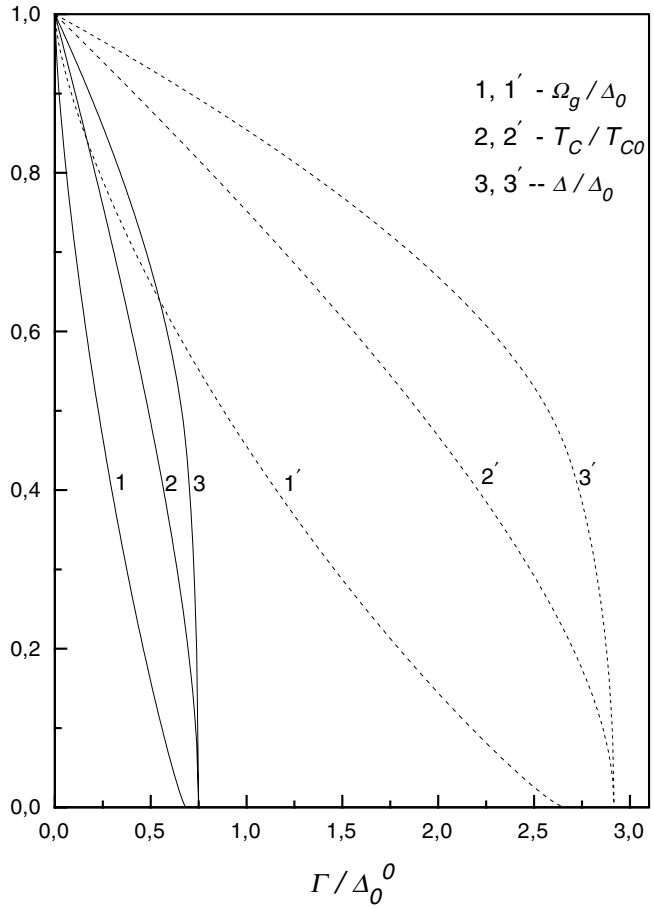


Figure 3. The critical temperature T_C, the order parameter Δ, and the energy gap Ω_G at $T = 0$ plotted as a function of the inverse relaxation time Γ for the adiabatic systems (1–3) and nonadiabatic ones (1′–3′) at $m = 1$.

with the above theory concerning the high-T_C compounds ($\omega_D \sim E_F = 0.1$ eV). For example, they prove the possibility of α changing from the value 0.25 to 1.5 (see, for example, curve 5 in figure 2). The value of $\alpha = 0.6$ at $\Gamma/2\pi T_0 = 0.4$ and $m = 1$ corresponds to the value of this quantity in $La_{1.85}Sr_{0.15}Cu_{1-x}M_x$ at $x \approx 0.11$ and $T_C = 30$ K [38].

The dependence of the quantities T_C/T_{C0}, Δ/Δ_0 and Ω_g/Δ_0 ($\Delta_0^0 = \Delta_0|_{m=0}$) as a function of impurity concentration is shown in figure 3 on the basis of solving the respective equations. Full lines correspond to the case of $m = 0$ and the broken ones to the case of $m = 1$. This figure gives the possibility of comparing the behaviour of the above mentioned quantities in ordinary superconductors (curves 1–3) with the case of high-T_C materials (curves 1′–3′). From this figure we can clearly see that the field of the gapless state $\Gamma_{cr} - \Gamma_{\Omega_g=0} = 0.066\Delta_0(0)$ at $m = 0$ and $0.257\Delta_0(0)$ at $m = 1$ arises in high-T_C materials in comparison with the case of ordinary superconductors.

From the studies performed above we can conclude that the nonadiabaticity effects and strong electron correlations enforce the superconductivity. In doped systems with paramagnetic impurity these factors favour weakening the destructive influence of the exchange interaction.

For a simple model of an isotropic superconductor in the weak coupling approximation ($\omega_0 \gg T_C$) and at low concentrations of the impurity ($\Gamma_C/\omega_0, \Gamma/E \ll 1$) we obtain:

(a) The equations for the quantities T_C (36), Δ (47) and Ω (52) have the same form as in the theory of normal superconductors with paramagnetic impurity [26, 30]. The main difference is in the considerably increased renormalization of coupling of the electron–phonon interaction λ at low values of the transferred momentum $Q_c \ll 2k_F$. This smallness is determined by the presence of strong electron correlations in the system caused by Coulomb interactions between electrons. The increase of λ in its turn increases the temperature of the superconducting transition T_{C0} (37) and order parameter Δ_0 (50). Renormalization of the parameter ρ (38), which is essentially dependent on the Migdal parameter $m = 2\omega_0/E_F$, decreased with the m increase in comparison with the case of normal superconductors ($m = 0$). As a result we obtain the smaller reduction of relations T_C/T_{C0}, Δ/Δ_0 and Ω_G/Δ_0 with increasing concentration of impurity, in contrast to normal superconductors (see figures 1 and 3).

(b) The coefficient of the isotope effect α decreases with increase of the parameter m and increases with increasing concentration of the impurity (figure 2). In this case the increase of the quantity α is weakened contrary to the case of normal superconductors. Low values of α are shown to be achieved in pure substances and these values can be remarkably increased in doped substances. At $m = 1$, for instance, we obtain $\alpha = 0.25$ at $\Gamma/2\pi T_0 = 0$ and $\alpha = 0.6$ at $\Gamma/2\pi T_0 \approx 0.6$ (curve 5, figure 2).

(c) The critical concentration of impurity Γ_{cr} at which superconductivity is destroyed as a function of the parameter m has a maximum at the point $m = 0.085$ (see figure 1). At $m \approx 1$, characterizing high-T_C systems, the critical concentration Γ_{cr} can also achieve values considerably greater than in normal superconductors due to the large values of T_{C0} and $f_c(m) < 1$ (39). At $m \sim 1$ the concentration of the impurity is $\Gamma_{\Omega_G=0}$ at which the gapless superconductivity begins (see figure 3). The Migdal parameter m (53) influences considerably this concentration of impurity because of the large values of Δ_0 and $f_c(m) < 1$. The range of gapless superconductivity $\Gamma_{cr} - \Gamma_{\Omega_G=0}$ is also increased due to the effects of nonadiabaticity.

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